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LETTER TO THE EDITOR

Forest fires as critical phenomena

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Abstract. A simple model is presented which describes the propagation of a fire in a densely packed forest from a centrally located burning tree. This model shares some features with random bond percolation from which it is distinct. We find that the critical probability, p_c is $\sim\frac{1}{2}$ for the triangular lattice, i.e. the onset of a spanning or 'penetrating' fire. In addition we find (i) a fractal dimensionality, d_f of 1.75 ± 0.05, (ii) the radius of gyration scales with time ($\bar{v}$) as 0.75 ± 0.02 and (iii) the number of burnt trees scales with time as $d_t\sim1.33\pm0.03$ where d_t is a metric exponent. More realistic models are presented for future consideration.

Percolation theory has been applied to a wide range of physical phenomena ranging from the evolution of galaxies to the spread of disease in an orchard (e.g. see Deutscher *et al* 1982, for a recent review of percolation and some areas of application). We shall in this letter consider the simplest model for a forest fire which is in fact, closely related to the General Epidemic process considered by Grassberger (1983) and Cardy (1983) and is also quite similar to a model proposed by Ritzenberg and Cohen (1984) to study the spread of electrical activity in the heart. The model of a forest fire analysed by Clavin *et al* (1983) may be considered as the high probability limit of this model on a random lattice of occupied trees.

Forest fires are not uncommon in North America and due to their destructive properties they have been carefully monitored and a wealth of information is available (Davis 1959). A fire may be typified by the following properties; its rate of spread, direction of travel and its intensity. We consider here low intensity fires (roughly 860 Kcal s^{-1} per metre of fire front) where from a geometrical standpoint, these fires may be considered 'thin' or two dimensional. Convection effects are minimal and the spread of the fire is a localised surface phenomenon. A high intensity or 'blow-up' fire (16 000–25 000 Kcal s^{-1} m^{-1}) is a three-dimensional phenomenon where strong convection currents lead to surface winds directed to the burning centre and burning embers propagate vast distances via the hot convection currents to create new centres of fire. Here we have both surface (or nearest-neighbour propagation) and long range hopping (through the embers).

The rate of spread of the fire is dependent on many constraints; adequate oxygen supply, wind velocity (these two factors lead to the faster spread of a fire in a direction *opposite* to the wind velocity than the case with no wind), topography (fires travel much faster up an incline and rarely down slope), age and type of trees (thick barks are more flame resistant) and most important, the recent rainfall. We shall consider a densely packed lattice (i.e. all the sites occupied by trees identical in terms of moisture content, age, etc.) and the spread of the fire is a localised surface phenomenon, i.e. a burning tree is only able to ignite its nearest neighbours. The fire commences with the central site ablaze at $t=0$ and a probability is assigned to the propagation of the fire to neighbouring 'warm' trees. The burning system leads to four categories of sites at any given time, t: (a) sites with burnt trees, M; (b) sites with burning trees N (c); warm trees (unburnt trees that are nearest neighbours to burning trees) and (d) remaining sites. A Monte Carlo time step consists of the propagation of the fire from ignited or burning trees, category (b), to warm trees, category (c) with probability p. At the end of the time step all the previously burning trees are considered dead or burnt and the warm trees that were ignited are now the new burning centres which create in turn a new set of warm trees. This set of warm trees may include unignited warm trees from earlier time steps. The process is continued until there are no further ignited trees (referred to as 'exhaustion' by Clavin *et al*) or the fire reaches the edge of the lattice ('penetration'). We note that an unburnt region surrounded by burnt or vacant sites cannot be ignited in this model.

We have simulated the spread of the forest fire on a triangular lattice of size $L\times L$ (L up to a 1000) and for a wide range of probabilities. We find that for high probabilities, $p \gtrsim 0.5$ the fire propagates as an expanding circle with very few unburnt trees and the number of ignited trees is proportional to the circumference of the circle. Lower values of p lead to a ramified cluster and for values of p less than $\sim\frac{1}{3}$ it is impossible to simulate a fire that penetrates or reaches the edge of the lattice. We expect that at p_c the total number of burnt trees, M to scale as (Havlin 1984, Hong and Stanley 1983)

$$M \sim t^{d_t} \tag{1}$$

where t is the number of Monte Carlo time steps and d_t is related to the topological exponent recently introduced by Hong *et al* (HHHS) (1984) to describe the number of sites within a chemical distance. The chemical distance describes the burnt sites relationships to the origin of the fire, e.g. the burnt trees at $t=3$ have a chemical distance of 3 etc. In addition we define the exponent $\bar{v}$ as

$$R_g \sim t^{\bar{v}} \tag{2}$$

where R_g is the radius of gyration of the burnt trees. The fractal dimensionality, d_f of the cluster of burnt trees is

$$M \sim R_g^{d_f} \tag{3}$$

where

$$d_f = d_t/\bar{v}. \tag{4}$$

Figure 1 shows burnt clusters of trees for $p>p_c$, $p\sim p_c$, $p<p_c$. Figure 2 shows the number of burning trees, N as a function of time, for several values of p. For $p>p_c$, N increases linearly with time, whereas for $p<p_c$, N increases initially but eventually falls to zero. At p_c the number of burning trees increases from 1 at $t=0$ and remains fairly constant from $t=200$ to $t=500$. Figure 3 shows several plots. (i) This is a plot of log M against log t for 5000 simulations of the forest fire on a 1000×1000 lattice. d_t, from this slope is 1.33 ± 0.05. This value is close to the value of 1.44 ± 0.03 reported by HHHS from their investigations of a fire on a percolating backbone. Their system may be considered as a forest on the sites of the backbone formed by the incipient infinite cluster with all the other sites of the lattice empty. A fire is started at a central site and all the neighbours on the backbone at $t=1$ will be ignited (i.e. $p=1$). (ii)

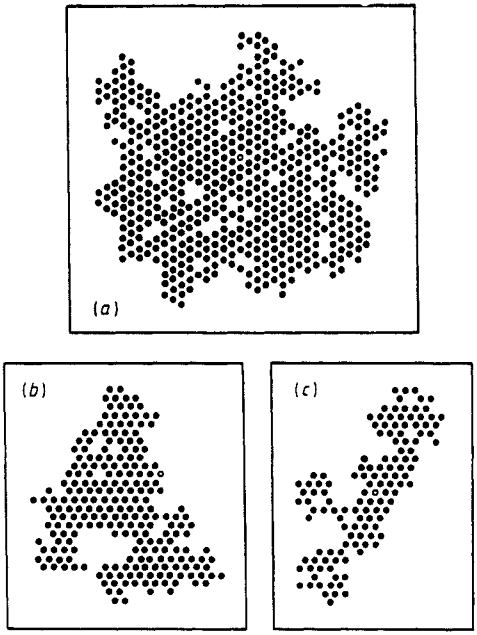

Figure 1. Typical clusters are shown for (*a*) $p(=0.40)>p_c$, (*b*) $p(=\frac{1}{3})\sim p_c$ and (*c*) $p(=0.30)<p_c$. Only finite clusters are formed for $p<p_c$. Note that at $p=0.40$ the cluster is fairly compact. The open circle (○) represents the origin of the fire in each case.

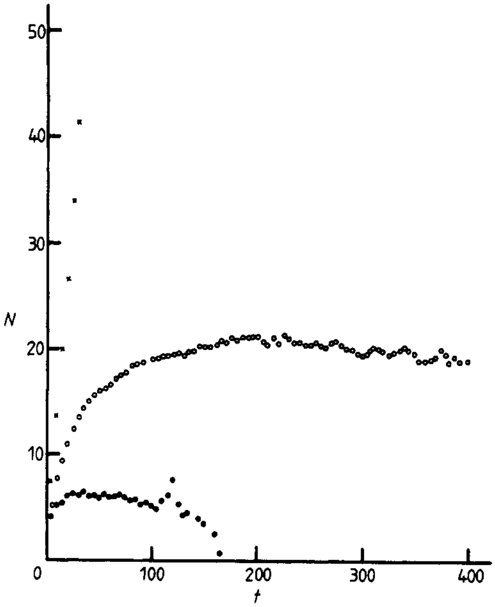

Figure 2. Shows the number of burning trees, N as a function of time, t for various values of p. Values of $p>p_c$ indicate that the number of burning trees increases linearly with time (×) whereas the number of burning trees tend to zero for p less than p_c (●). For $p\sim p_c$ we find that the number of burning trees remain fairly constant (○).

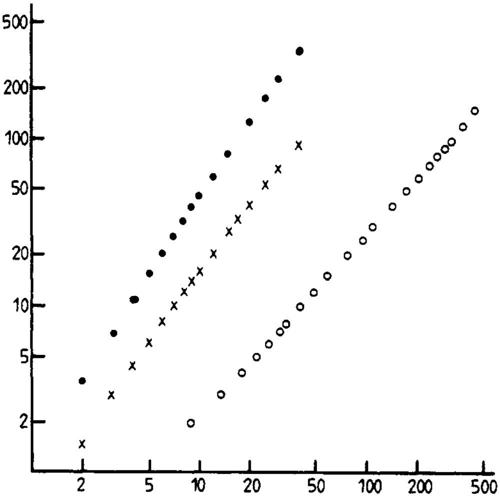

Figure 3. The figure shows the variation of (i) radius of gyration, R_g against time, t; (ii) number of burnt trees, M against time; (iii) radius of gyration, R_g against mass of trees, M; represented by ●, ×, ○, respectively.

This shows the variation of R_g with time and $\bar{v}$ is 0.75 ± 0.02. This value is distinct from that reported by HHHS, where $\bar{v} = 0.87 \pm 0.02$ for their model. (iii) This shows the variation of R_g with M and from which we obtain a slope of 1.75 ± 0.05 for the fractal dimensionality of the cluster of burnt trees which is in good agreement for the value 1.77 obtained through equation (4).

A more extensive analysis of the model is in progress where we are investigating the effects of vacancies on p_c as well as on the critical exponents. In addition, we would like to include the effects of wind velocity, terrain and to take into account different species of trees.

In summary, we have introduced a model which is appropriate for the propagation of a low intensity fire. We find a critical probability for the triangular lattice of $\frac{1}{3}$, a fractal dimensionality of 1.75 ± 0.05, a metric dimensionality d_t of 1.33 ± 0.05 and an exponent $\bar{v}$ of 0.78 ± 0.03.

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